Thermal Radiation Effects on MHD Flow of Nanofluid over an Exponentially Stretching Sheet with Heat and Mass Fluxes

[1]P. Roja, [2]T. Sankar Reddy & [3]M. Gnaneswara Reddy
[1] Department of Mathematics, Annamacharya Institute of Technology and sciences, Rajampet, Kadapa-516126, A.P., India.
[2] Department of Mathematics, Annamacharya Institute of Technology and sciences, C. K. Dinne, Kadapa-516003, A.P., India.
[3] Department of Mathematics, Acharya Nagarjuna University Campus, Ongole -523 001, A.P., India.

ABSTRACT:

This research work addresses the thermal radiation effects on magneto hydrodynamics (MHD) flow of an incompressible nano fluid due to an exponentially stretching sheet with heat and mass fluxes boundary conditions. Similarity transformations are used to obtain the self-similar equations which are then solved numerically using shooting technique along with fourth order Runge-Kutta method. Characteristics of various sundry parameters on the non-dimensional velocity, temperature, nanoparticle volume fraction, local Nusselt and Sherwood numbers are visualized. Besides these the numerical values of skin friction coefficient, local Nusselt and Sherwood numbers are also computed and analyzed.

Keywords: *Thermal Radiation; Magneto hydrodynamics (MHD); Nanofluid; Heat and Mass fluxes.*

---

1. INTRODUCTION:

In recent years, the analysis of flow and heat transfer over a stretching surface have achieved extensive attention because of its wide applications, such as continuous casting, exchangers, metal spinning, bundle wrapping, foodstuff processing, chemical processing, equipment and polymer extrusion. Crane [1] was the first who study the Newtonian fluid flow caused by a stretching sheet. Many researchers Dutta *et al*. [2], Chen and Char [3] and Gupta [4] modified the work of Crane [1] by taking the effect of mass transfer under various circumstances. Nadeem *et al*. [5] took the exponential stretching sheet to discuss the heat transfer phenomenon of water-based nanofluid. Mukhopadhyay *et al*. [6] scrutinized the heat transfer flow over a porous exponential stretching sheet with thermal radiation. Zhang *et al*. [7] concentrates the heat transfer of the power law nanofluid thin film occur due to a stretching sheet in the presence of velocity slip effect and magnetic field. The boundary layer flow of ferromagnetic fluid over a stretching surface is demonstrated by Majeed *et al*. [8]. Pal and Saha [9] examined the unsteady stretching sheet to discuss the heat and mass transfer in a thin liquid film with the effect of non linear thermal radiation. Weidman [10] studied a unified formulation for stagnation point flows with stretching surfaces.

The study of magnetohydrodynamics (MHD) flow of an electrically conducting fluid over a stretching sheet has promising applications in modern metallurgical as well as in metal-working procedures. Many professional techniques regarding polymers require the cooling of continuous strips and filaments by drawing them from moving fluid. The final product depends greatly on the rate of cooling that is governed by the structure of the boundary layer close to the stretching sheet. Mukhopadhyay *et al*. [11] studied MHD flow of Casson fluid due to exponentially stretching sheet with thermal radiation. Impact of magnetohydrodynamics in bidirectional flow of nanofluid subject to second order slip velocity and homogeneous–heterogeneous reactions is reported by Hayat et al. [12]. Lin *et al*. [13] examined unsteady MHD pseudo-plastic nanofluid flow and heat transfer in a finite thin film over stretching surface with internal heat generation. Sheikholeslami *et al*. [14] analyzed the effect of thermal radiation on magnetohydrodynamics nanofluid flow and heat transfer by means of two phase model. Application of the HAM-based Mathematica package BVP h 2.0 on MHD Falkner–Skan flow of nanofluid is provided by Farooq et al. [15]. Shehzad et al. [16] presented an analytical study to investigate thermal radiation effects in three-dimensional flow of Jeffrey nanofluid with internal heat generation and magnetic field.

The importance of radiation cannot be ignored in the processes that are performed at very high temperature. The radiative effects are also important in gas turbines, missiles, aircraft, space vehicles and nuclear power plants. The interaction of radiation in thermally convective flow of viscous liquid over an inclined surface is made by Moradi et al. [17]. Sheikholeslami et al. [18] addressed the effect of radiation in viscous nanofluid flow by considering the two phasemodel. Laminar flow of an Oldroyd-B liquid with nanoparticles and radiation is examined by Hayat et al. [19]. Ashraf et al. [20] studied the radiative three dimensional flow of Maxwell fluid with thermophoresis and convective condition. Hayat et al. [21] reported the thermal radiation in laminar flow of Powell-Eyring nanofluid over a stretching cylinder. Further, the convective conditions are more useful and realistic in transpiration cooling process, material drying etc. Aziz [22] implemented the convective condition in boundary layer flow of viscous fluid past a flat plate. Hayat et al. [23] studied the effects of Joule heating and thermophoresis in stretched flow of Maxwell model under convective condition. Sakiadis flow of Maxwell fluid with convective boundary condition is discussed by Mustafa et al. [24]. Hayat et al. [25] analytically discussed the stagnation point flow of Maxwell fluid in the presence of thermal radiation and convective condition. Hayat et al. [26] examined inclined magnetic field and heat source/sink aspects in flow of nanofluid with nonlinear thermal radiation. Nonlinear radiative flow of three-dimensional Burgers nanofluid with new mass flux effect is scrutinized by Khan et al. [28].

In the present paper, the thermal radiation effects magnetohydrodynamic (MHD) flow of an incompressible nano fluid due to an exponentially stretching sheet with heat and mass fluxes conditions is analyzed. Using similarity transformations, the governing partial differential equations are transformed into the self-similar ordinary differential equations which are then solved numerically by the shooting method.

2. FORMULATION:

We consider the two-dimensional hydro magnetic flow of an incompressible fluid by an exponentially stretching sheet. Heat and mass transfer analysis is characterized in the presence of thermal radiation, heat source/sink and chemical reaction. A non-uniform magnetic field $B(x) =$

$B_0 exp(x/2l)$ is applied in the y-direction. Induced magnetic field for small magnetic Reynolds number is neglected. We imposed the heat and mass fluxes boundary conditions at the surface of the sheet. The governing equations of motion may be written as:

(i) Continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{1}$$

(ii) Momentum:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

(iii) Energy:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{(\rho c)_p}{(\rho c)_f}\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{3}$$

(iv) Nanoparticle volume fraction:

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=D_B\frac{\partial^2 N}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

subject to the boundary conditions:

$$u=U_w(x)=\mathrm{U}_0\exp\left(\frac{x}{l}\right),\ v=-V(x),$$
$$\frac{\partial T}{\partial y}=-\frac{q_w(x)}{\alpha},\ \frac{\partial N}{\partial y}=-\frac{q_{np}(x)}{D_B},\ \ at\ y=0 \tag{5a}$$

$$u\to 0,\ T\to T_\infty,\ N\to N_\infty,\ \text{as}\ y\to\infty \tag{5b}$$

Here u and v denote the velocity components in the x and y directions respectively, ν the kinematic viscosity, $\alpha=\frac{k}{\rho c_p}$ the thermal diffusivity, k the fluid density, ρ the thermal conductivity, c_p the specific heat, T the fluid temperature, T_∞ the ambient temperature, N the fluid concentration, C_∞ the ambient concentration, $\alpha=k/\rho c_p$ the thermal diffusivity, k the thermal conductivity, c_p the specific heat, $q_r=\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial T}{\partial Y}$ the radiative heat flux, k^* the mean absorption coefficient, σ^* the Stefan-Boltzmann constant, $(\rho c)_p$ the effective heat capacity of nanoparticles, $(\rho c)_f$ heat capacity of the base fluid. N is nanoparticle volume, D the mass diffusion $U_w(x)=\mathrm{U}_0\exp(x/l)$ is the stretching velocity of sheet, U_0 the reference velocity, l the reference length, $q_w(x)=q_{w0}\,\mathrm{T}_0\sqrt{U_0/2\nu l}\,\exp(x/l)$ the variable heat flux,

$q_{np}(x) = q_{np0}C_0\sqrt{U_0 / 2\nu l}\exp\left(x/l\right)$ the variable surface nanoparticle flux, U_0, T_0, q_{w0}, q_{np0}, N_0, are the reference velocity, temperature and heat flux, surface nanoparticle flux, nanoparticle volume fraction respectively, $V(x) = V_0\exp\left(x/l\right)$ a special type of velocity at the wall is considered (Bhattacharyya [28]) where V_0 is a constant. Here $V(x) > 0$ is the velocity of suction and $V(x) < 0$ is the velocity of injection.

Introducing similarity transformations as follows:

$$\eta = y\left(\frac{U_0}{2\nu x}\right)^{1/2}\exp\left(\frac{x}{l}\right),\ \psi = \left(2\nu U_0 x\right)^{1/2} f\left(\eta\right)\exp\left(\frac{x}{l}\right),$$

$$u = U_0 f'\left(\eta\right)\exp\left(\frac{x}{l}\right),\ v = -\sqrt{\frac{\nu U_0}{2l}}\exp\left(\frac{x}{l}\right)\left[f\left(\eta\right) - \eta f'\left(\eta\right)\right], \tag{6}$$

$$T = T_\infty + \frac{q_{w0}}{\alpha}T_0\exp\left(\frac{x}{l}\right)\theta\left(\eta\right), C = C_\infty + \frac{q_{np0}}{\alpha}C_0\exp\left(\frac{x}{l}\right)\phi\left(\eta\right)$$

If the dimensional stream function $\psi(x, y)$ then $u = \dfrac{\partial\psi}{\partial y}$ and $u = -\dfrac{\partial\psi}{\partial x}$.

The continuity equation is automatically satisfied and using similarity transformation, the system of Eqs. (2), (3) and (4) becomes:

$$f''' + ff'' - 2f'^2 - Ha^2 f' = 0 \tag{7}$$

$$\left(1 + \frac{4}{3}R\right)\theta'' + \Pr\left(f\theta' + f'\theta + N_b\theta'\phi' + N_t\theta'^2\right) = 0 \tag{8}$$

$$\phi'' + Le(f\phi' - f'\phi) + \frac{N_t}{N_b}\theta'' = 0 \tag{9}$$

Here primes mean differentiation with respect to η, $Ha = \dfrac{\sigma B_0^2(x)l}{\rho U_w(x)}$ is the Hartmann number, $\Pr = \dfrac{\nu}{\alpha}$ is the Prandtl number, $R = \dfrac{4\sigma^* T_\infty^3}{kk^*}$ is the radiation parameter and $Le = \dfrac{\nu}{D_B}$ is the Lewis number, $N_b = \dfrac{\left(\rho c\right)_p q_{np0}}{\left(\rho c\right)_f \nu}N_0$ is the Brownian motion parameter and $N_t = \dfrac{D_T}{T_\infty}\dfrac{\left(\rho c\right)_p q_{w0}}{\left(\rho c\right)_f \alpha\nu}T_0$ is the thermophoresis parameter, respectively.

The transformed boundary conditions (5a) and (5b) are given by

$$\begin{aligned} &f(0) = S,\ f'(0) = -1,\ \theta(0) = -1,\ \phi(0) = -1 \\ &f'(\infty) = 0,\ \theta(\infty) = 0,\ \phi(\infty) = 0 \end{aligned} \tag{10}$$

Where $S = \dfrac{-v_0}{\sqrt{\nu c / 2l}}$ is suction/injection parameter. Here the parameter is positive S>0 (v_0<0) for mass suction and negative S<0 (v_0>0) for mass injection.

The physical quantities of interest are the local skin friction coefficient, the wall heat transfer coefficient (or the local Nusselt number) and the wall deposition flux (or the local Stanton number) which are defined as respectively where the skin friction C_f, the heat transfer $q_w(x)$ and the mass transfer Sh_x from the wall are given by

$$\sqrt{2C_f \operatorname{Re}_x} = f''(0),\ C_f = \frac{u}{U_w \exp(x/l)}\left(\frac{du}{dy}\right)_{y=0}, \tag{11}$$

From the temperature field, we can study the rate of heat transfer which is given by

$$\frac{Nu_x}{\sqrt{\operatorname{Re}_x}} = -\sqrt{\frac{x}{2l}}\left(1+\frac{4}{3}R\right)\theta'(0),\ Nu_x = -\frac{x}{(T_w - T_\infty)}\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{12}$$

From the concentration field, we can study the rate of mass transfer which is given by

$$\frac{Sh_x}{\sqrt{\operatorname{Re}_x}} = -\sqrt{\frac{x}{2l}}\phi'(0),\ Sh_x = -\frac{x}{(C_w - C_\infty)}\left(\frac{\partial C}{\partial y}\right)_{y=0} \tag{13}$$

where $\operatorname{Re}_x = U_0 x/\nu$ the local Reynolds number.

3. METHOD OF SOLUTION:

The system of ordinary differential equations (7) – (9) subject to the boundary conditions (10) are solved numerically using Runge–Kutta fourth-order integration with shooting technique. A step size of $\Delta\eta = 0.01$ was selected to be satisfactory for a convergence criterion of 10^{-6} in all cases. The results are presented graphically in Figs. (1)–(6) and conclusions are drawn for flow field and other physical quantities of interest that have significant effects.

4. RESULTS AND DISCUSSION:

For the illustration of the results, Eqs. (7) –(9) with boundary conditions (10) are solved numerically by Runge–Kutta fourth-order integration with shooting method and numerical values are plotted in Figs. (2) – (6). The leading parameters are keeping fixed as *Ha*=1.0, *S* = 3.0, *Le* = 1.3, *R* = 0.1, Pr = 0.71, *Nt* = 0.8 and *Nb* = 0.5 throughout the computations. The influence of the involving parameters Hartmann number *Ha*, Lewis number *Le*, Radiation parameter *R*, thermophoresis number *Nt*, suction parameter *s* and Brownian motion parameter *Nb* on the velocity, temperature and nanoparticle volume friction profiles. Figs.1(a)-(c), respectively, illustrate the velocity, temperature and nanoparticle volume friction profiles for various values of suction parameter *S*. From Fig. 1(a) , the velocity profiles increase with increasing in suction parameter. It is also observed Fig. 1(b) that the temperature decreases when suction parameter increases. Further, from Fig. 1(c), it is found that nanoparticle volume friction decreases as suction parameter increases.

The effect of the Hartmann number (i.e. magnetic field parameter *Ha*) on the velocity, temperature and nanoparticle volume friction profiles are presented in Figs 2(a)-(c), respectively. We observe from Fig. 2(a) that the velocity profiles increase with increasing values of Hartmann number. Physically by increasing magnetic field the Lorentz force increases. More resistance is offered to the motion of fluid and thus the velocity of the fluid is increased. It is also seen Fig. 2(b) that the temperature decreases as Hartmann number increases. In addition, from Fig. 2(c) it is found that nanoparticle volume fraction profile increases, as Hartmann number increases.

Fig. 3(a)-(b) display the effects due to thermophoresis parameter *Nt* on temperature and nanoparticle volume fraction are represented. Due to increase of thermophoresis parameter, both the temperature (Fig. 3(a)) and nanoparticle volume fraction (Fig. 3(b)) profiles increase. Thermophoresis parameter *Nt* is the ratio of the nanoparticle diffusion to the thermal diffusion in the nanofluid. Due to increase in *Nt* the temperature difference between the sheet and the fluid increases and as a result thermal boundary layer increases in this case. With the increase in *Nt*, thermophoresis force increases which helps the nanoparticle to move from hot to cold regions. Due to this movement nanoparticle volume fraction increases.

Fig. 4(a)-(b) depict the influence of radiation parameter *R* on temperature and nanoparticle volume fraction profiles. It is noted that larger values of *R* enhances the temperature profile. This is due to the reason that an increase in *R* corresponds to smaller mean absorption coefficient. We observe from Fig. 4(b) that as *R* increases the nanoparticle volume fraction profile increases.

Finally, Fig(5) and (6) demonstrate the effects of Lewis number *Le* and Brownian motion parameter N_b on the nanoparticle volume fraction profiles, respectively. It is observed from Fig. (5) that nanoparticle volume fraction distribution decreases as Lewis number increases. This is probably because of the fact that an increase in *Le* results in smaller Brownian diffusion coefficient D_B which restricts nanoparticles to infiltrate deeper into fluid. Consequently, a thinner nanoparticle volume fraction occurs for a higher Lewis number *Le*. Moreover, the reduction is occurs in nanoparticle volume fraction profile with increasing values of Brownian motion parameter N_b. This may result in the thickening of thermal boundary layer. Physically, a rise in Brownian motion causes an increase in the diffusion of nanoparticles which reduces the concentration inside the boundary layer.

Numerical data of the influences of various parameters of interest on Nusselt number and Sherwood number are studied in Table 1. Tabulated values clearly indicate that the value of Nusselt number increases by increasing *R* while it decreases with an increase in the values of *Ha* and *S*. On the other hand, Sherwood number increases by increasing the values of *Ha* and *R*, but opposite behaviors for higher *S*.

Table 1: Numerical values of local Nusselt number and local Sherwood number for different values of *Ha, R* and *S* when *Ha*=1.0, *Nt*=0.8, *Nb*=0.5, *Pr*=0.71, *R*=0.1 and *Le* = 1.3.

Parameters(fixed values)	Parameters	$\mathrm{Re}_x^{-1/2}\, Nu_x$	$\mathrm{Re}_x^{-1/2}\, Sh_x$
Nt=0.8, Nb = 0.5, S=3.0, Pr=0.71, R=0.1, Le=1.3	Ha=1.0	0.451145	0.294788
Nt=0.8, Nb = 0.5, S=3.0, Pr=0.71, R=0.1, Le=1.3	1.5	0.438589	0.287824
Nt=0.8, Nb = 0.5, S=3.0, Pr=0.71, R=0.1, Le=1.3	3.0	0.403746	0.267842
Nt=0.8, Nb = 0.5, S=3.0, Pr=0.71, Ha=1.0, Le=1.3	R=0.10	0.460557	0.381022
Nt=0.8, Nb = 0.5, S=3.0, Pr=0.71, Ha=1.0, Le=1.3	0.15	0.466654	0.377892
Nt=0.8, Nb = 0.5, S=3.0, Pr=0.71, Ha=1.0, Le=1.3	0.30	0.467774	0.377317
Nt=0.8, Nb = 0.5, R=0.1, Pr=0.71, Ha=1.0, Le=1.3	S=0.5	0.415307	0.251433
Nt=0.8, Nb = 0.5, R=0.1, Pr=0.71, Ha=1.0, Le=1.3	0.6	0.423403	0.262577
Nt=0.8, Nb = 0.5, R=0.1, Pr=0.71, Ha=1.0, Le=1.3	0.8	0.440726	0.278440

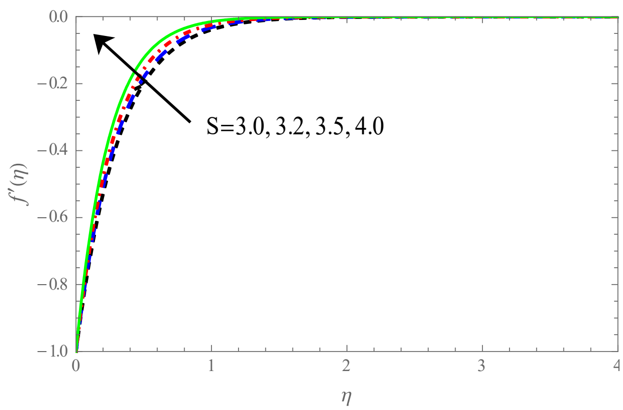

Fig 1(a). Effect of S on $f'(\eta)$

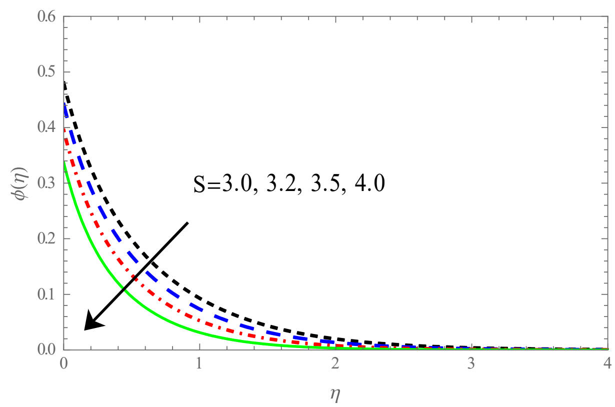

Fig 1(c). Effect of S on $\phi(\eta)$

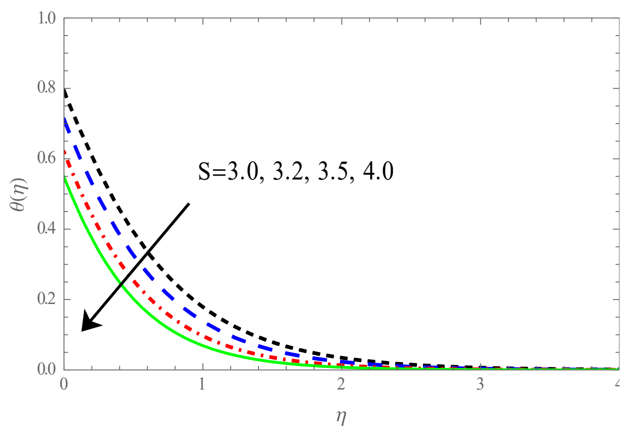

Fig 1(b). Effect of S on $\theta(\eta)$

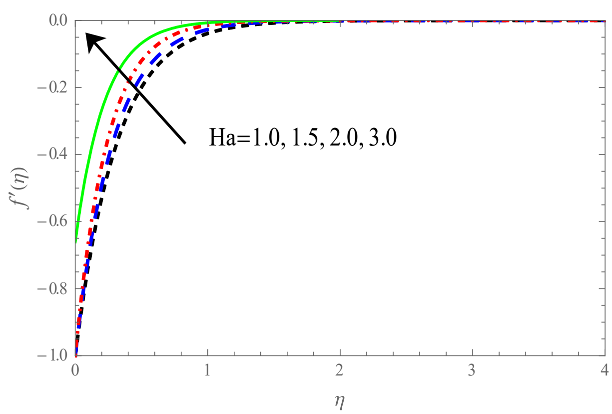

Fig 2(a). Effect of Ha on $f'(\eta)$

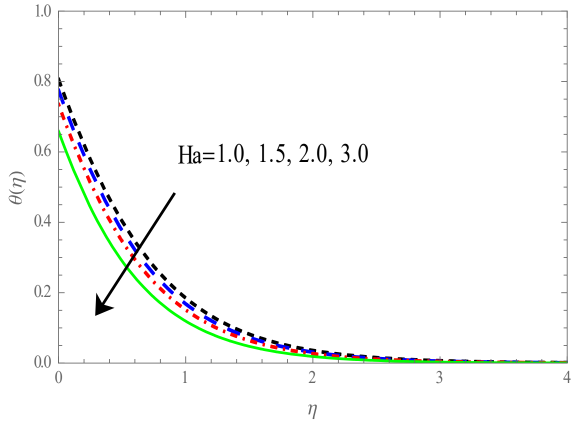

Fig 2(b). Effect of Ha on $\theta(\eta)$

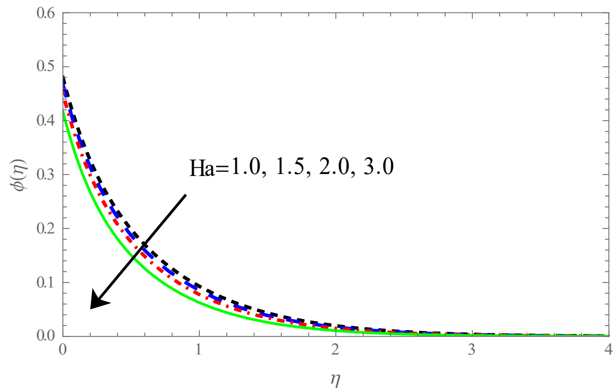

Fig 2(c). Effect of Ha on $\phi(\eta)$

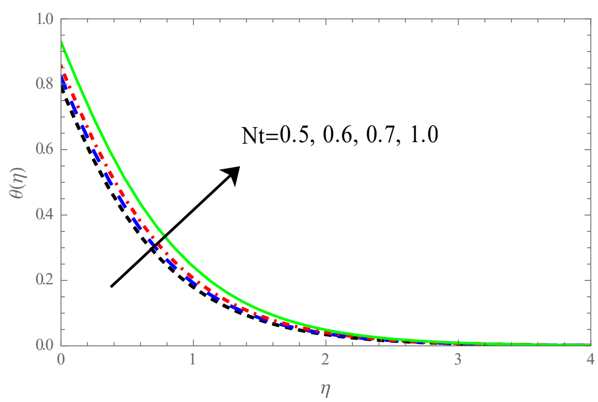

Fig 3(a). Effect of Nt on $\theta(\eta)$

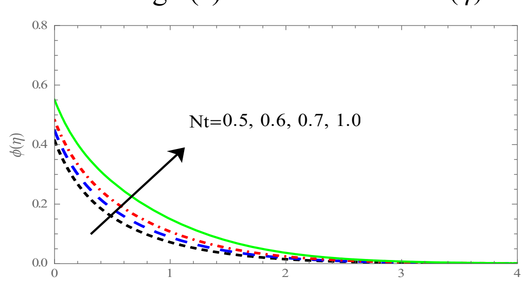

Fig 3(b). Effect of Nt on $\phi(\eta)$

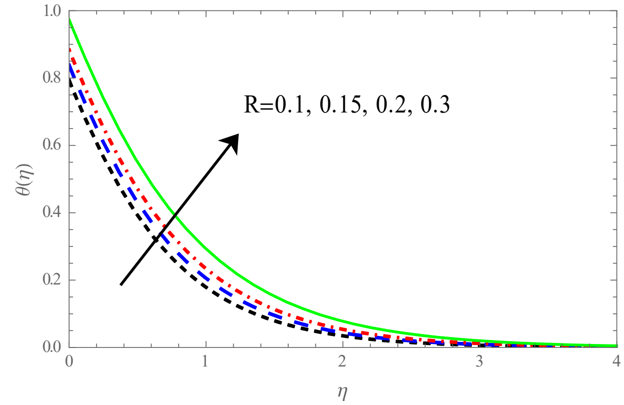

Fig 4(a). Effect of R on $\theta(\eta)$

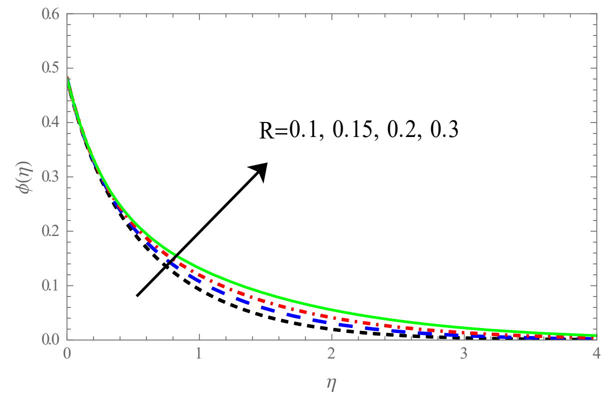

Fig 4(b). Effect of R on $\phi(\eta)$

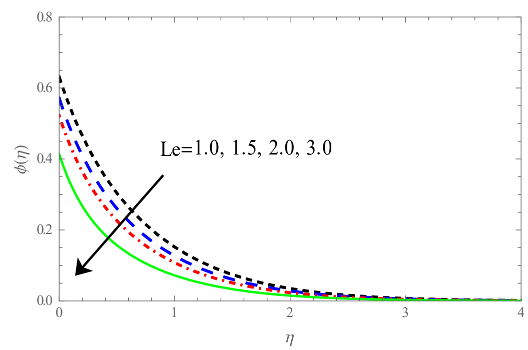

Fig 5. Effect of Le on $\phi(\eta)$

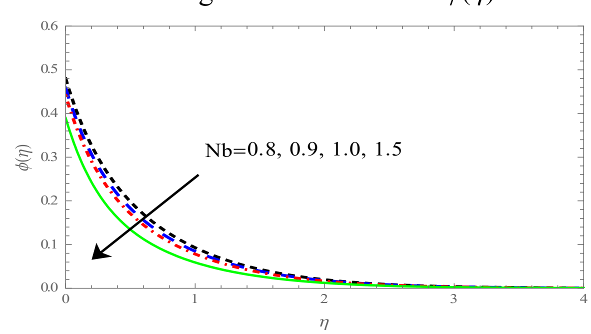

Fig 6. Effect of Nb on $\phi(\eta)$

CONCLUSION:

Combined effects of thermal radiation and Magnetohydrodynamics in flow of nanofluid and heat and mass transfer analysis by an exponentially starching sheet with heat and mass flux conditions have been examined. The governing partial differential equations have been rendered into a set of nonlinear coupled, ordinary differential equations using suitable transformations and the resulting well-posed boundary value problem has been solved numerically using the Runge–Kutta fourth order based shooting method. Effects of pertinent parameters on flow, temperature and nanoparticle volume friction fields are discussed with graphical illustrations. From the present study, the main conclusions may be summarized as follows:

(i) Velocity profile and boundary layer thickness increase via mixed convection parameter k.
(ii) Temperature field hðgÞ yields a decrease via larger Prandtl number
(iii) The nanoparticle volume fraction increases as the value of squeeze parameter decreases.
(iv) Higher values of ratio parameter A results in the reduction of temperature profile and enhancement in local Nusselt number.
(v) The nanoparticle volume fraction increases as the value of squeeze parameter decreases.
(vi) Local Sherwood number is increasing function of n; A; Sc and c

REFERENCES:

[1] Crane, L .J.,(1970). Flow past a stretching plate. Zeitschrift für angewandte Mathematik und Physik ZAMP, 21(4), 645–647.

[2] Dutta, B. K, Roy P, Gupta, A.S.(1985). Temperature field in flow over a stretching sheet with uniform heat flux. Int Commun Heat Mass Transfer ,12(1):89–94.

[3] Char, M.I. (1988). Heat transfer of a continuous, stretching surface with suction or blowing. J Math Anal Appl., 135(2):568–80.

[4] Gupta ,P.S., and Gupta, A.S., (1977). Heat and mass transfer on a stretching sheet with suction or blowing. Can J Chem Eng, 55(6):744–746.

[5] Nadeem S, Haq R.U., and Khan Z .H.,(2014). Heat transfer analysis of water-based nanofluid over an exponentially stretching sheet. Alexandria Eng. J., 53(1) pp. 219–224.

[6] Mukhopadhya S.,(2013). Slip effects on MHD boundary layer flow over an exponentially stretching sheet with suction/blowing and thermal radiation. Ain Shams Eng J., 4(3):485–491.

[7] Zhang, Y. Zhang, M, Bai Y.,(2017). Unsteady flow and heat transfer of power-law nanofluid thin film over a stretching sheet with variable magnetic field and power-law velocity slip effect. J Taiwan Inst Chem Eng., 70:104–10.

[8] Majeed A, Zeeshan A, Ellahi R (2016). Unsteady ferromagnetic liquid flow and heat transfer analysis over a stretching sheet with the effect of dipole and prescribed heat flux. J Mol Liq 223:528–233.

[9] Pal D, Saha P (2016). Influence of nonlinear thermal radiation and variable viscosity on hydromagnetic heat and mass transfer in a thin liquid film over an unsteady stretching surface. Int J Mech Sci, Vol. 119:208–216.

[10] Weidman P, Turner M R (2017). Stagnation-point flows with stretching surfaces: a unified formulation and new results. Eur J Mech B Fluids, 61, 144–153.

[11] S. Mukhopadhyay, I.C. Moindal, T. Hayat(2014). MHD boundary layer flow of Casson fluid passing through an exponentially stretching permeable surface with thermal radiation, Chin. Phys. B, 23 104701-12.

[12] T. Hayat, M. Imtiaz, A(2015). Alsaedi, Impact of magnetohydrodynamics in bidirectional flow of nanofluid subject to second order slip velocity and homogeneous–heterogeneous reactions, J. Magn. Magn. Mater., 395 294–302.

[13] Y. Lin, L. Zheng, X. Zhang, L. Ma, G. Chen(2015). MHD pseudo-plastic nanofluid unsteady flow and heat transfer in a finite thin film over stretching surface with internal heat generation, Int. J. Heat Mass Transf., 84 903–911.

[14] M. Sheikholeslami, D.D. Ganji, M.Y. Javed, R. Ellahi (2015). Effect of thermal radiation on magnetohydrodynamics nanofluid flow and heat transfer by means of two phase model, J. Magn. Magn. Mater., 374, 36–43.

[15] U. Farooq, Y.L. Zhao, T. Hayat, A. Alsaedi, S.J. Liao (2015). Application of the HAM-based Mathematica package BVPh 2.0 on MHD Falkner–Skan flow of nanofluid, Comput. Fluids, 11 69–75.

[16] S.A. Shehzad, Z. Abdullah, A. Alsaedi, F.M. Abbaasi, T. Hayat (2016). Thermally radiative three-dimensional flow of Jeffrey nanofluid with internal heat generation and magnetic field, J. Magn. Magn. Mater., 397, 108–114.

[17] A. Moradi, H. Ahmadikia, T. Hayat, A. Alsaedi(2013). On mixed convection radiation interaction about an inclined plate through a porous medium, Int. J. Thermal Sci. 64,129–136.

[18] M. Sheikholeslami, D.D. Ganji, M.Y. Javed, R. Ellahi (2015). Effect of thermal radiation on magnetohydrodynamics nanofluid flow and heat transfer by means of two phase model, J. Mag. Magnetic Materials 374, 36–43.

[19] Hayat T., T. Hussain, S. Shehzad, A. Alsaedi, A (2015). Flow of Oldroyd-B fluid with nanoparticles and thermal radiation, Appl. Math. Mech. 36, 69-80.

[20] M. Ashraf, T. Hayat, S. Shehzad, A. Alsaedi(2015) Mixed convection radiative flow of three dimensional Maxwell fluid over an inclined stretching sheet in presence of thermophoresis and convective condition, AIP Adv. 5, 027134.

[21] Hayat T., N. Gull, M. Farooq, B. Ahmad (2015). Thermal radiation effect in MHD flow of Powell-Eyring nanofluid induced by a stretching cylinder, J. Aerospace Eng., ASCE. 29(1),1943-5525.

[22] Aziz, A.(2009) similarity solution for laminar thermal boundary layer over a flat plate with a convective surface boundary condition, Commun. Nonlin. Sci. Numer. Simul. 14 1064–1068.

[23] Hayat T., M. Waqas, S.A. Shehzad, A. Alsaedi (2014). Effects of Joule heating and thermophoresis on stretched flow with convective boundary conditions, Scientia Iranica B 21, 682–692.

[24] M. Mustafa, J. Khan, T. Hayat, A. Alsaedi(2015). Sakiadis flow of maxwell fluid considering magnetic field and convective boundary conditions, AIP Adv., 5, 027106.

[25] Hayat T., M. Waqas, S. Shehzad, A. Alsaedi(2013). Mixed convection radiative flow of Maxwell fluid near a stagnation point with convective condition, J. Mech. 29, 403–409.

[26] Hayat T., S. Qayyum, A. Alsaedi, A. Shafiq(2016). Inclined magnetic field and heat source/sink aspects in flow of nanofluid with nonlinear thermal radiation, Int. J. Heat Mass Transfer 103, 99–107.

[27] Khan, M. Khan, W.A., Alshomrani, A.S. (2016). Non-linear radiative flow of three dimensional Burgers nanofluid with new mass flux effect, Int. J. Heat Mass Transfer, 101, 570–576.

[28] Bhattacharyya, K., (2011)., Boundary layer flow and heat transfer over an exponentially shrinking sheet, Chin. Phys. Lett., 28(7), 4701.